

Flavor origin of R-parity

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Proton stability is guaranteed in the MSSM by assuming a discrete symmetry, R-parity. However, there are additional R-parity conserving higher dimensional operators which violate lepton and baryon numbers and induce fast proton decay. Here we study the possibility that all renormalizable, as well as the most dangerous non-renormalizable, R-parity violating operators are forbidden by a flavor symmetry, providing a common origin for fermion mixing and proton and dark matter stability. We propose a specific model based on the $\Delta(27)$ discrete symmetry.

In July 2012, the ATLAS [1] and CMS [2] collaborations announced the discovery of a new boson, with a mass in the 126 GeV ballpark. Although detailed studies of its properties need to be done in order to confirm its identity, one is tempted to interpret this new particle in terms of the long-awaited Higgs boson. In any case, this constitutes a decisive breakthrough in high energy physics that, once correctly understood, will surely shed light on the dynamics of the electroweak scale. The discovery of the Higgs boson, and the measurement of its mass, reminds us a long-standing theoretical problem in particle physics: the famous hierarchy problem [3]. This can be expressed as the high sensitivity that fundamental scalars have to physics at high energies. Unless one accepts a very precise fine-tuning of the parameters of the theory, the Higgs mass is naturally pushed to those high energies, destabilizing the electroweak scale. Supersymmetry (SUSY) is one of the most popular solutions to the hierarchy problem. If the SUSY breaking scale is low (not far from the TeV scale), the electroweak scale is kept under control by virtue of the cancellations between bosonic and fermionic contributions to the Higgs mass.

When constructing a supersymmetric model one finds new gauge and SUSY invariant renormalizable interactions, not present in the standard model, that lead to lepton (L) and baryon (B) number violation. With the particle content of the Minimal Supersymmetric Standard Model (MSSM), these are

$$[LH_u]_F, [LLe]_F, [LQd]_F, [udd]_F, \quad (1)$$

where F stands for F-terms. If they were simultaneously present in the lagrangian, the proton would have a fast decay rate unless very small coefficients are introduced. For that reason, one usually introduces a discrete symmetry called R-parity, $R_p = (-1)^{3(B-L)+2s}$ (where s is

the spin of the particle), that forbids the L and B violating terms shown above [4]. The conservation of R-parity has very relevant phenomenological implications. This discrete symmetry stabilizes the lightest supersymmetric particle (LSP). As a consequence of that, supersymmetric events at colliders contain large amounts of missing energy in the final state. Furthermore, if neutral, the LSP would be the perfect example of a Weakly Interacting Massive Particle (WIMP) and a good dark matter candidate. R-parity has, however, some drawbacks. First of all, R-parity is introduced by hand in the MSSM, without a theoretical explanation for its origin. And second, there are additional non-renormalizable interactions which, even though they break lepton and/or baryon numbers, are perfectly allowed by R-parity. Even if these operators are generated at the Planck scale, they would lead to unacceptably fast proton decay unless their coefficients are tiny [5].

Many theoretical ideas have been proposed in order to explain the origin of R-parity. Most of them consider R-parity as a remnant after the breaking of a larger symmetry group, see for example [6]. However, in most cases there is no explanation for the suppression of the higher-dimensional operators. On the other hand, the origin of fermion masses hierarchies and mixings, the so-called flavor problem, is another long-standing mystery in particle physics. One interesting possibility to address the flavor problem is the introduction of an horizontal symmetry between the three generations of fermions. The flavor symmetry, which can be either continuous or discrete, imposes some structures on the Yukawa couplings.

In this letter we propose that R-parity may be a consequence of the flavor symmetry. In this case, quark and lepton mixings, as well as the stability of proton and dark matter, can be explained in a common framework.

Let us consider the MSSM extended by a $\Delta(27)$ flavor symmetry, see for instance [7]. This discrete group is a subgroup of $SU(3)$ (for a classification see [8]) that belongs to the series $\Delta(3n^2)$. It has 11 irreducible representations, namely two triplets $\mathbf{3}, \mathbf{3}^*$ and 9 singlets $\mathbf{1}_i$. The product rules are $\mathbf{3} \times \mathbf{3}^* = \sum_{i=1}^9 \mathbf{1}_i$ and $\mathbf{3} \times \mathbf{3} =$

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	\hat{L}	\hat{e}	\hat{H}_d	\hat{H}_u	\hat{Q}	\hat{d}	\hat{u}
$\Delta(27)$	3	1_{1,2,3}	3*	3	3	1_{1,2,3}	3

TABLE I: $\Delta(27)$ charges of the MSSM superfields.

$\mathbf{3}^* + \mathbf{3}^* + \mathbf{3}^*$. From these rules, it is clear that the product $\mathbf{3} \times \mathbf{3} \times \mathbf{3}$ is invariant under $\Delta(27)$ whereas $\mathbf{3} \times \mathbf{3} \times \mathbf{3}^*$ is not. We assign $\Delta(27)$ representations to the MSSM particle content as shown in Table I. It is straightforward to check that this assignment forbids all the aforementioned R-parity violating couplings. Therefore, R-parity results as an accidental symmetry originated by the underlying flavor symmetry of the model. Furthermore, the flavor symmetry also forbids the most dangerous non-renormalizable operators (those with lower dimensions, $d = 5$) that break L or B numbers. These are

$$\begin{aligned}
\mathcal{O}_1^{(5)} &= [QQQL]_F & \mathcal{O}_2^{(5)} &= [uude]_F \\
\mathcal{O}_3^{(5)} &= [QQQH_d]_F & \mathcal{O}_4^{(5)} &= [QueH_d]_F \\
\mathcal{O}_5^{(5)} &= [LLH_uH_u]_F & \mathcal{O}_6^{(5)} &= [LH_dH_uH_u]_F \\
\mathcal{O}_7^{(5)} &= [H_uH_ue^*]_D & \mathcal{O}_8^{(5)} &= [H_u^*H_de]_D \\
\mathcal{O}_9^{(5)} &= [QuL^*]_D & \mathcal{O}_{10}^{(5)} &= [ud^*e]_D
\end{aligned}$$

Here, we include F-terms that may be present in the superpotential and D-terms which may be present in the Kähler potential. They are denoted with the subscripts F and D , respectively. These operators appear in the lagrangian with a mass suppression $1/\Lambda$, where Λ is the energy scale associated to the L and/or B number violating physics beyond the MSSM. Even if this scale is taken as large as the Planck scale, these dangerous operators would lead to too fast proton decay if the corresponding coefficients are of order one. Since all these operators are forbidden by $\Delta(27)$, we conclude that, regarding proton stability, the flavor symmetry makes a better job than R-parity¹.

One could similarly list all dimension six ($d = 6$) non-renormalizable operators that induce lepton and/or baryon number violation [10]. This list is, of course, much longer. Although $\Delta(27)$ forbids many of them, some are allowed. A simple example is $\mathcal{O}^{(6)} = [uddLH_u]_F$, which breaks both lepton and baryon numbers. These operators have a mass suppression $1/\Lambda^2$ and thus they are less dangerous than the $d = 5$ ones, only requiring relatively small coefficients. Finally, the $\Delta(27)$ symmetry allows the usual three fermion Yukawa couplings, as well as the μ term. Then, we conclude that one recovers the MSSM superpotential (with the restrictions imposed by

the flavor symmetry),

$$\mathcal{W}_{\text{MSSM}} = Y_u \hat{Q} \hat{H}_u \hat{u} + Y_d \hat{Q} \hat{H}_d \hat{d} + Y_l \hat{L} \hat{H}_l \hat{e} + \mu \hat{H}_u \hat{H}_d, \quad (2)$$

where we omit the $\Delta(27)$ contractions. Note, however, that three possible $\mathbf{3} \times \mathbf{3}$ contractions (to a $\mathbf{3}^*$ representation) exist. Therefore, each Yukawa coupling in Eq. (2) should be understood as three different parameters, accounting for the three possible $\Delta(27)$ invariant products. We denote them as $Y_{u,d,l}^{1,2,3}$. The textures for the charged fermion mass matrices can be obtained easily from the rules introduced above. The results read

$$M_{l,d} \sim \begin{pmatrix} Y_{l,d}^1 \langle H_d^1 \rangle & Y_{l,d}^2 \langle H_d^1 \rangle & Y_{l,d}^3 \langle H_d^1 \rangle \\ Y_{l,d}^1 \langle H_d^2 \rangle & \omega Y_{l,d}^2 \langle H_d^2 \rangle & \omega^2 Y_{l,d}^3 \langle H_d^2 \rangle \\ Y_{l,d}^1 \langle H_d^3 \rangle & \omega^2 Y_{l,d}^2 \langle H_d^3 \rangle & \omega Y_{l,d}^3 \langle H_d^3 \rangle \end{pmatrix} \quad (3)$$

$$M_u \sim \begin{pmatrix} Y_u^1 \langle H_u^1 \rangle & Y_u^2 \langle H_u^2 \rangle & Y_u^3 \langle H_u^1 \rangle \\ Y_u^1 \langle H_u^2 \rangle & Y_u^1 \langle H_u^2 \rangle & Y_u^2 \langle H_u^3 \rangle \\ Y_u^2 \langle H_u^1 \rangle & Y_u^3 \langle H_u^3 \rangle & Y_u^1 \langle H_u^3 \rangle \end{pmatrix} \quad (4)$$

with $\omega^3 = 1$. It is not difficult to show that by taking the vacuum expectation value (VEV) alignment $\langle H_{u,d}^0 \rangle \sim (1, 1, 1)$, which breaks $\Delta(27)$ into a Z_3 subgroup, all the charged fermion mass matrices are diagonalized on the left by the same unitary matrix, the so-called *magic* matrix U_ω , defined as

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \quad (5)$$

Then, each charged fermion mass matrix in Eqs. (3) and (4) can be fitted to reproduce the corresponding three fermion masses (this can be easily done thanks to the three different Yukawas couplings for each sector) and the CKM mixing matrix turns out to be proportional to the identity. This is usually regarded as a good starting point when building a model. As next step one can consider a completely broken $\Delta(27)$, breaking the alignment $\langle H_{u,d}^0 \rangle \sim (1, 1, 1)$, and leading to a CKM that deviates from the identity. Similarly, the CKM matrix can also be generated at the loop level from the flavor structure associated with the SUSY breaking terms [11]. Other possibilities to generate the CKM mixing could be by means of adding extra mirror quarks, scalar-mediated interactions [12] or by using different singlets of $\Delta(27)$ [13].

So far, we have only discussed the viability of the framework. Unfortunately, the model resulting from the addition of the $\Delta(27)$ discrete symmetry is just the MSSM, with some restrictions in the parameters². Therefore, we do not expect any new collider signature

¹ Other alternatives to R-parity can also forbid (or strongly suppress) higher-dimensional operators with lepton and/or baryon number violation, see for example [9].

² To be precise, the only difference with the canonical MSSM is the existence of three Higgs doublets. This may lead to a very rich phenomenology in flavor physics.

	$\hat{\Sigma}_1$	$\hat{\Sigma}_2$	\hat{S}
$\Delta(27)$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}^*$

TABLE II: Charge assignment of the additional superfields in the extended model for neutrino masses. The superfields $\hat{\Sigma}_{1,2}$ are triplets of $SU(2)_L$ with $Y = 0$ and \hat{S} is a neutral $SU(2)_L$ singlet.

that is not present in the canonical MSSM. In order to find new predictions one needs to extend the model in order to account for neutrino masses. The new structures must preserve $\Delta(27)$ as well, and this leads to interesting consequences.

As shown before, the flavor symmetry forbids the Weinberg operator, $\mathcal{O}_5^{(5)} = [LLH_uH_u]_F$. Therefore, in order to generate neutrino masses one is forced to go beyond minimal models and consider higher dimensional operators. Let us consider a gauge singlet chiral superfield, \hat{S} , in the $\mathbf{3}^*$ representation of $\Delta(27)$. Then, the operator

$$\mathcal{O}^{(6)} = [LLH_uH_uS]_F \quad (6)$$

is allowed by all symmetries. We give now a realization of the operator in Eq. (6). To the particle content in Table I, we add the superfields in Table II. The singlet \hat{S} should not be confused with the NMSSM singlet superfield. In fact, note that the $\hat{S}\hat{H}_u\hat{H}_d$ superpotential term is forbidden by $\Delta(27)$. Besides \mathcal{W}_{MSSM} defined in Eq. (2) the superpotential contains

$$\mathcal{W} \supset Y_\Sigma \hat{L}\hat{H}_u\hat{\Sigma}_1 + M\hat{\Sigma}_1\hat{\Sigma}_2 + \lambda\hat{S}\hat{\Sigma}_2\hat{\Sigma}_2 + \kappa_S\hat{S}^3 \quad (7)$$

Other superpotential terms are forbidden by the gauge and flavor symmetries³. When the scalar component of \hat{S} gets a VEV, an effective Majorana mass for the Σ_2 triplet (the fermionic component of the $\hat{\Sigma}_2$ superfield) is generated. This leads to an inverse seesaw mechanism [14] induced by $SU(2)_L$ triplets (for other realizations of the *inverse type-III seesaw* see [15]). In the basis $\psi^T = (\nu, \Sigma_1^0, \Sigma_2^0)$, we obtain the 9×9 mass matrix for the neutral fermions

$$M_\nu = \begin{pmatrix} 0 & Y_\Sigma \langle H_u \rangle & 0 \\ Y_\Sigma^T \langle H_u \rangle & 0 & M \\ 0 & M^T & \lambda \langle S \rangle \end{pmatrix}, \quad (8)$$

which, assuming $\lambda v_S \ll Y_\Sigma v_u \ll M$, leads to

$$m_\nu = v_u^2 v_S Y_\Sigma (M^T)^{-1} \lambda M^{-1} Y_\Sigma^T \quad (9)$$

where $v_u = \langle H_u^0 \rangle$ and $v_S = \langle S \rangle$. In fact, it is worth emphasizing some advantages that our model has with

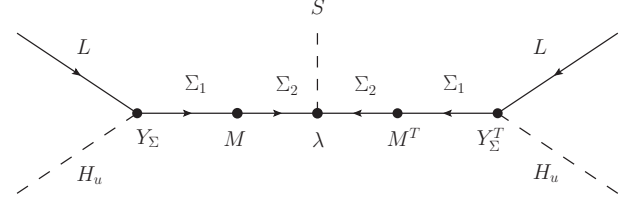


FIG. 1: $\mathcal{O}^{(6)} = [LLH_uH_uS]_F$ realization.

respect to the conventional inverse seesaw. Typically, in the context of inverse seesaw models it is quite difficult to forbid the $\Sigma_1^0 \Sigma_1^0$ mass term after one allows for lepton number violation. Here its absence is a direct consequence of the $\Delta(27)$ symmetry. Moreover, in order to have new physics at the TeV scale, that is $M \sim \text{TeV}$, and $\mathcal{O}(1)$ Yukawa couplings, the parameter $\mu = \lambda v_S$ must be of the order of 10 eV. In the original inverse seesaw mechanism the μ parameter is expected to be naturally small in the 't Hooft sense since the limit $\mu \rightarrow 0$ enhances the symmetry of the lagrangian (lepton number is recovered). Here this is not the case, but μ can be dynamically suppressed by means of the VEV of the scalar S . Note that lepton number is explicitly broken by the trilinear superpotential term $\kappa_S \hat{S}^3$ and thus no majoron, the Goldstone boson associated to the spontaneous breaking of $U(1)_L$, appears in the spectrum. However, R-parity remains unbroken after the addition of this term, since S carries two units of lepton number. The operator in Eq. (6) is obtained after integrating out the $\hat{\Sigma}_{1,2}$ superfields, as depicted in Figure (1). Although the dimension 5 operator $\mathcal{O}^{(5)} = [uddS]_F$, which breaks baryon number, is in principle allowed after the introduction of the new superfield \hat{S} , it is not generated at tree-level since \hat{S} does not couple to the quark superfields.

Let us now discuss the resulting neutrino mixing pattern. Using the $\Delta(27)$ contraction rules, one finds that the mass matrix M is proportional to the identity matrix. The Dirac neutrino mass matrix $m_D = Y_\Sigma \langle H_u^0 \rangle$ and the matrix $\mu = \lambda \langle S \rangle$ have the structures

$$m_D = \begin{pmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{pmatrix}, \quad \mu = \begin{pmatrix} \alpha' & \beta' & \beta' \\ \beta' & \alpha' & \beta' \\ \beta' & \beta' & \alpha' \end{pmatrix} \quad (10)$$

when all the scalar fields take VEV in the $(1,1,1)$ direction. In this case, the charged lepton mass matrix is diagonalized by the magic matrix U_ω , see Eq. (5). One can now perform a U_ω rotation in order to go to the basis where the charged lepton mass matrix is diagonal. In this basis, the neutrino mass matrix can be written as

$$\tilde{m}_\nu \sim U_\omega m_\nu U_\omega^T \sim \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix} \quad (11)$$

where the parameters a and b are functions of $\alpha, \beta, \gamma, \alpha', \beta', \gamma'$. In general $a \neq b$. By setting $a = b$ we

³ We note that if the $\hat{\Sigma}_{1,2}$ were singlets under $SU(2)_L$, superpotential terms $\kappa_{\Sigma_i} \hat{\Sigma}_i^3$ would be allowed, thus breaking R-parity explicitly.

recover the result of the Babu-Ma-Valle model [16]. In this limit, the mass matrix in Eq. (11) gives maximal atmospheric angle and degenerate neutrino mass spectrum, while the solar and reactor mixing angles are zero. As observed in [16], Eq. (11) is corrected by wave-function renormalizations of ν_e , ν_μ , and ν_τ , as well as the corresponding vertex renormalizations, lifting the neutrino degeneracy and the solar/reactor mixing angles. The resulting neutrino mass matrix can be written as

$$\tilde{m}_\nu^{1\text{-loop}} \sim \begin{pmatrix} a(1 + 2\delta_{ee}) & a\delta_{e\mu} + b\delta_{e\tau}^* & b\delta_{e\mu}^* + a\delta_{e\tau} \\ & 2b\delta_{\mu\tau}^* & b(1 + \delta_{\mu\mu} + \delta_{\tau\tau}) \\ & & 2b\delta_{\mu\tau} \end{pmatrix},$$

where δ_{ij} parametrize the radiative corrections. When $a = b$ (as in [16]) and assuming real δ_{ij} corrections, the resulting neutrino mass matrix is $\mu - \tau$ symmetric giving maximal atmospheric angle and zero θ_{13} reactor angle. The solar angle is a free parameter and can be fitted. If the corrections are allowed to be complex a non-zero value for θ_{13} can be obtained. In this case, CP violation is predicted to be maximal [16]. Regarding the nature of the corrections, these come from flavor mixing in the slepton/sneutrino sector. A detailed study can be found in [17]. Note that our model has more freedom since $a \neq b$ in general. This can be used to relax some of the restrictions in the parameter space. Nevertheless, large $\tilde{\mu} - \tilde{\tau}$ mixing is necessary, typically predicting $\text{Br}(\tau \rightarrow \mu\gamma)$ close to its experimental limit. Furthermore, in the SUSY inverse seesaw one expects large Z-penguin contributions

in lepton flavor violating processes [18]. Therefore, observables such as $\text{Br}(\tau \rightarrow \mu\ell\ell)$, with $\ell = e, \mu$, are also expected to set important constraints on the SUSY parameter space. Finally, violation of lepton flavor universality in observables such as R_K and R_π is also an important test of the model [19].

Regarding collider phenomenology, the $\Sigma_{1,2}$ $SU(2)_L$ triplet fermions can be pair produced at the LHC due to their gauge interactions. This possibility, not present in singlet extensions of the MSSM, may lead to very clear signatures with additional leptons and/or lepton flavor violation in the $\Sigma_{1,2}$ decays [20]. It is well-known that the assumption of R-parity in the MSSM serves to stabilize the LSP. Here we obtain the same result, R -parity, without assuming it. Therefore, the usual MSSM candidate, a neutralino LSP, can play the role of the dark matter of the universe.

In summary, we have proposed a SUSY model based on the $\Delta(27)$ flavor symmetry. All lepton and/or baryon number violating operators of dimension 4 and 5 are forbidden by the flavor symmetry, thus providing a single explanation for the proton and dark matter stabilities.

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